α-Discounting Method for Multi-Criteria Decision Making (α-D MCDM)

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Abstract **– In this paper we introduce a new procedure called α-Discounting Method for Multi-Criteria Decision Making (α-D MCDM), which is as an alternative and extension of Saaty's Analytical Hierarchy Process (AHP). It works for any set of preferences that can be transformed into a system of homogeneous linear equations. A degree of consistency (and implicitly a degree of inconsistency) of a decision-making problem are defined. α-D MCDM is generalized to a set of preferences that can be transformed into a system of linear and/or non-linear homogeneous and/or non-homogeneous equations and/or inequalities. Consistent, weak consistent, and strong consistent examples are presented in the sequel for linear and non-linear decision making problems.**

Keywords: Multi-Criteria Decision Making (MCDM), Analytical Hierarchy Process (AHP), α-Discounting Method, Fairness Principle, parameterize, pairwise comparison, n-wise comparison, consistent MCDM problem, weak or strong inconsistent MCDM problem

1 Introduction

α-Discounting Method for Multi-Criteria Decision Making (α-D MCDM) is an alternative and extension of Saaty's Analytical Hierarchy Process (AHP). It works not only for preferences that are pairwise comparisons of criteria as AHP does, but for preferences of any n-wise (with $n \geq 2$) comparisons of criteria that can be expressed as linear homogeneous equations.

The general idea of α-D MCDM is to assign null-null positive parameters $\alpha_1, \alpha_2, \ldots, \alpha_p$ to the coefficients in the right-hand side of each preference that diminish or increase them in order to transform the above linear homogeneous system of equations which has only the null-solution, into a system having solutions.

After finding the general solution of this system, the principles used to assign particular values to all parameters α's is the second important part of α-D, yet to be deeper investigated in the future. In the current paper we herein propose the Fairness Principle, i.e. each coefficient should be discounted with the same percentage (we think this is fair: not making any favouritism or unfairness to any coefficient), but the reader can propose other principles.

For consistent decision-making problems with pairwise comparisons, α-Discounting Method together with the Fairness Principle give the same result as AHP.

But for weak inconsistent decision-making problem, α-Discounting together with the Fairness Principle give a different result from AHP.

α-Discounting/Fairness-Principle together give a justifiable result for strong inconsistent decision-making problems with two preferences and two criteria; but for more than two preferences with more than two criteria and the Fairness Principle has to be replaced by another principle of assigning numerical values to all parameters α's.

Since Saaty's AHP is not the topic of this paper, we only recall the main steps of applying this method, so the results of α-D MCDM and of AHP could be compared.

AHP works for only for pairwise comparisons of criteria, from which a square Preference Matrix, A (of size $n \times n$), is built. Then one computes the maximum eigenvalue $\lambda_{\max}$ of A and its corresponding eigenvector.

If $\lambda_{\max}$ is equal to the size of the square matrix, then the decision-making problem is consistent, and its corresponding normalized eigenvector (Perron-Frobenius vector) is the priority vector.

If $\lambda_{\max}$ is strictly greater than the size of the square matrix, then the decision-making problem is inconsistent. One raise to the second power matrix A, and again the resulted matrix is raised to the second power, etc. obtaining the sequence of matrices A^2, A^4, A^8, …, etc. In each case, one computes the maximum eigenvalue and its associated normalized eigenvector, until the difference between two successive normalized eigenvectors is smaller than a given threshold. The last such normalized eigenvector will be the priority vector.

Saaty defined the Consistency Index as:

$$\mathrm{CI}(A) = \frac{\lambda_{\max}(A) - n}{n-1},$$

where n is the size of the square matrix A.

2 α-Discounting Method for Multi-Criteriaia Decision Making (α-D MCDM)

2.1 Description of α-D MCDM

The general idea of this paper is to discount the coefficients of an inconsistent problem to some percentages in order to transform it into a consistent problem.

Let the Set of Criteria be $C = \{C_1, C_2, \ldots, C_n\}$, with $n \geq 2$, and the Set of Preferences be $P = \{P_1, P_2, \ldots, P_m\}$, with $m \geq 1$.

Each preference P_i is a linear homogeneous equation of the above criteria $C_1, C_2, \ldots, C_n$:

$$P_i = f(C_1, C_2, \ldots, C_n).$$

We need to construct a basic belief assignment (bba):

$$m: C \rightarrow [0, 1]$$

such that $m(C_i) = x_i$, with $0 \leq x_i \leq 1$, and

$$\sum_{i=1}^{n} m(x_i) = 1.$$

We need to find all variables x_i in accordance with the set of preferences P.

Thus, we get an m□n linear homogeneous system of equations whose associated matrix is

$A = (a_{ij})$, $1 \leq i \leq m$ and $1 \leq j \leq n$.

In order for this system to have non-null solutions, the rank of the matrix A should be strictly less than n.

2.2 Classification of Linear Decision-Making Problems

a) We say that a **linear decision-making problem is consistent** if, by any substitution of a variable x_i from an equation into another equation, we get a result in agreement with all equations.

b) We say that a **linear decision-making problem is weakly inconsistent** if, by at least one substitution of a variable x_i from an equation into another equation, we get a result in disagreement with at least another equation in the following ways:

(WD1) $$\left\{ \begin{array}{l} x_i = k_1 \cdot x_j, k > 1; \\ x_i = k_2 \cdot x_j, k_2 > 1, k_2 \neq k_1 \end{array} \right\}$$

or

(WD2) $$\left\{ \begin{array}{l} x_i = k_1 \cdot x_j, 0 < k < 1; \\ x_i = k_2 \cdot x_j, 0 < k_2 < 1, k_2 \neq k_1 \end{array} \right\}$$

or

(WD3) $$\left\{ x_i = k \cdot x_i, k \neq 1 \right\}$$

(WD1)-(WD3) are weak disagreements, in the sense that for example x > y always, but with different ratios (like x=3y and x=5y).

All disagreements in this case should be like (WD1)-(WD3).

c) We say that a **linear decision-making problem is strongly inconsistent** if, by at least one substitution of a variable x_i from an equation into another equation, we get a result in disagreement with at least another equation in the following way:

(SD4) $$\left\{ \begin{array}{l} x_i = k_1 \cdot x_j; \\ x_i = k_2 \cdot x_j, \end{array} \right\} \text{with } 0 < k_1 < 1 < k_2$$

or $0 < k_2 < 1 < k_1$

(i.e. from one equation one gets $x_i < x_j$ while from the other equation one gets the opposite inequality: $x_j < x_i$).

At least one inconsistency like (SD4) should exist, no matter if other types of inconsistencies like (WD1)-(WD3) may occur or not.

Compute the determinant of A.

a) If det(A)=0, the decision problem is consistent, since the system of equations is dependent.
It is not necessarily to parameterize the system. {In the case we have parameterized, we can use the Fairness Principle – i.e. setting all parameters equal $\alpha_1 = \alpha_2 = \ldots = \alpha_p = \alpha > 0$}.
Solve this system; find its general solution.
Replace the parameters and secondary variables, getting a particular solution.
Normalize this particular solution (dividing each component by the sum of all components).
Wet get the priority vector (whose sum of its components should be 1).

b) If $\det(A) \neq 0$, the decision problem is inconsistent, since the homogeneous linear system has only the null-solution.
b1) If the inconsistency is weak, then parameterize the right-hand side coefficients, and denote the system matrix A(α).
Compute det(A(α)) = 0 in order to get the parametric equation.

If the Fairness Principle is used, set all parameters equal, and solve for $\alpha > 0$.
Replace α in $A(\alpha)$ and solve the resulting dependent homogeneous linear system.
Similarly as in a), replace each secondary variable by 1, and normalize the particular solution in order to get the priority vector.
b2) If the inconsistency is strong, the Fairness Principle may not work properly. Another approachable principle might by designed.
Or, get more information and revise the strong inconsistencies of the decision-making problem.

2.3 Comparison between AHP and α-D MCDM

a) α-D MCDM's general solution includes all particular solutions, that of AHP as well;
b) α-D MCDM uses all kind of comparisons between criteria, not only paiwise comparisons;
c) for consistent problems, AHP and α-D MCDM/Fairness-Principle give the same result;
d) for large inputs, in α-D MCDM we can put the equations under the form of a matrix (depending on some parameters alphas), and then compute the determinant of the matrix which should be zero; after that, solve the system (all can be done on computer using math software);
the software such as MATHEMATICA and APPLE for example can do these determinants and calculate the solutions of this linear system;
e) α-D MCDM can work for larger classes of preferences, i.e. preferences that can be transformed in homogeneous linear equations, or in non-linear equations and/or inequalities – see more below.

2.4 Generalization of α-D MCDM

Let each preference be expressed as a linear or non-linear equation or inequality. All preferences together will form a system of linear/non-linear equations/inequalities, or a mixed system of equations and inequalities.
Solve this system, looking for a strictly positive solution (i.e. all unknowns $x_i > 0$). Then normalize the solution vector.
If there are more such numerical solutions, do a discussion: analyze the normalized solution vector in each case.
If there is a general solution, extract the best particular solution.
If there is no strictly positive solution, parameterize the coefficients of the system, find the parametric equation, and look for some principle o apply in order to find the numerical values of the parameters α 's. A discussion might also be involved. We may get undetermined solutions.

3 Degrees of Consistency and Inconsistency in α-D MCDM/Fairness-Principle

For α-D MCDM/Fairness-Principle in consistent and weak consistent decision-making problems, we have the followings:

a) If $0 < \alpha < 1$, then α is the **degree of consistency** of the decision-making problem, and $\beta = 1-\alpha$ is the **degree of inconsistency** of the decision-making problem.
b) If $\alpha > 1$, then $1/\alpha$ is the **degree of consistency** of the decision-making problem, and $\beta = 1-1/\alpha$ is the **degree of inconsistency** of the decision-making problem.

4 Principles of α-D MCDM (Second Part)

a) In applications, for the second part of α-D Method, the Fairness Principle can be replaced by other principles.
Expert's Opinion. For example, if we have information that a preference's coefficient should be discounted twice more than another coefficient (due to an expert's opinion), and another preference's coefficient should be discounted a third of another one, then appropriately we set for example: $\alpha_1 = 2\alpha_2$ and respectively $\alpha_3 = (1/3)\alpha_4$, etc. in the parametric equation.
b) For α-D/Fairness-Principle or Expert's Opinion.
Another idea herein is to set a **threshold of consistency** t_c (or implicitly **a threshold of inconsistency** t_i). Then, if the degree of consistency is smaller than a required t_c, the Fairness Principle or Expert's Opinion (whichever was used) should be discharged, and another principle of finding all parameters α's should be designed; and similarly if the degree of inconsistency is bigger than t_i.
c) One may measure the system's accuracy (or error) for the case when all m preferences can be transformed into equations; for example, preference P_i is transformed into an equation $f_i(x_1, x_2, \ldots, x_n)=0$; then we need to find the unknowns $x_1, x_2, \ldots, x_n$ such that:

$$e(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{m} |f_i(x_1, x_2, \ldots, x_n)|$$ is minimum,

where "e" means error.
Calculus theory (partial derivatives) can be used to find the minimum (if this does exist) of a function of n variables, $e(x_1, x_2, \ldots, x_n)$, with $e: R_+^n \rightarrow R_+$.

For consistent decision-making problems the system's accuracy/error is zero, so we get the exact result.

We prove this through the fact that the normalized priority vector [a_1 a_2 … a_n], where x_i=a_i > 0 for all i, is a particular solution of the system $f_i(x_1, x_2, …, x_n)=0$ for i=1, 2, …, m; therefore:

$$\sum_{i=1}^{m} |f_i(a_1, a_2, ..., a_n)| = \sum_{i=1}^{m} | 0 | = 0.$$

But, for inconsistent decision-making problems we find approximations for the variables.

5 Extension of α-D MCDM (Non-Linear α-D MCDM)

It is not difficult to generalize the α-D MCDM for the case when the preferences are non-linear homogeneous (or even non-homogeneous) equations.

This non-linear system of preferences has to be dependent (meaning that its general solution – its main variables - should depend upon at least one secondary variable).

If the system is not dependent, we can parameterize it in the same way. Then, again, in the second part of this Non-Linear α-D MCDM we assign some values to each of the secondary variables (depending on extra-information we might receive), and we also need to design a principle which will help us to find the numerical values for all parameters. We get a particular solution (such extracted from the general solution), which normalized will produce our priority vector.

Yet, the Non-Linear α-D MCDM is more complicated, and depends on each non-linear decision-making problem.

6 Consistent Example 1

Let the Set of Preferences be {C1, C2, C3},
and the Set of Criteria be:

1. $C1$ is 4 times as important as $C2$.
2. $C2$ is 3 times as important as $C3$.
3. $C3$ is one twelfth as important as $C1$.

Let $m(C1) = x$, $m(C2) = y$, $m(C3) = z$.

6.1 Using α-Discounting MCDM

The linear homogeneous system associated to this decision-making problem is:

$$\begin{cases} x = 4y \\ y = 3z \\ z = \dfrac{x}{12} \end{cases}$$

whose associated matrix A_1 is:

$$\begin{pmatrix} 1 & -4 & 0 \\ 0 & 1 & -3 \\ -1/12 & 0 & 1 \end{pmatrix},$$

whence $\det(A_1) = 0$, so the DM problem is consistent.

Solving this homogeneous linear system we get its general solution that we set as a vector [12z 3z z], where z can be any real number (z is considered a secondary variable, while x=12z and y=3z are main variables).

Replacing z=1, the vector becomes [12 3 1], and then normalizing (dividing by 12+3+1=16 each vector component) we get the priority vector: [12/16 3/16 1/16], so the preference will be on C1.

6.2 Using AHP

We get the same result.

The preference matrix is:

$$\begin{pmatrix} 1 & 4 & 12 \\ 1/4 & 1 & 3 \\ 1/12 & 1/3 & 1 \end{pmatrix}$$

whose maximum eigenvalue is $\lambda_{\max}$ = 3 and its corresponding normalized eigenvector (Perron-Frobenius vector) is [12/16 3/16 1/16].

6.3 Using Mathematica 7.0 Software

Using MATHEMTICA 7.0 software, we graph the function:

$h(x,y) = |x-4y|+|3x+4y-3|+|13x+12y-12|$, with $x,y \in [0,1]$, which represents the consistent decision-making problem's associated system:

x/y=4, y/z=3, x/z=12, and x+y+z=1, x>0, y>0, z>0.

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In[1]:=
Plot3D[Abs[x-4y]+Abs[3x+4y-3]+Abs[13x+12y-
12],{x,0,1},{y,0,1}]
```

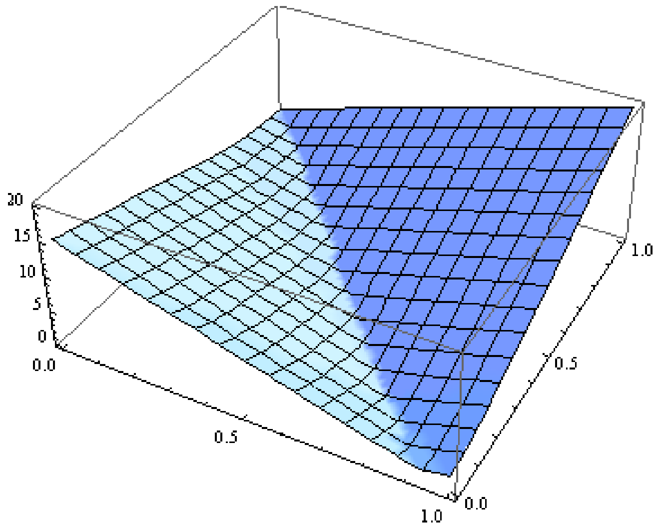

Fig. 1

The minimum of this function is zero, and occurs for x=12/16, y=3/16.

If we consider the original function of three variables associated with h(x,y) we have:

H(x,y,z) = |x-4y|+|y-3z|+|x-12z|, x+y+z=1, with x,y,z $\in$ [0,1],

we also get the minimum of H(x,y,z) being zero, which occurs for x=12/16, y=3/16, z=1/16.

7 Weak Inconsistent Example 2 where AHP Doesn't Work

Let the Set of Preferences be {C1, C2, C3}.

and the Set of Criteria be:

1. $C1$ is 2 times as important as $C2$ and 3 times as important as $C3$ put together.
2. $C2$ is half as important as $C1$.
3. $C3$ is one third as important as $C1$.

7.1 Let's use the α-D MCDM Method

Let $m(C1) = x$, $m(C2) = y$, $m(C3) = z$;

$$\begin{cases} x = 2y + 3z \\ y = \dfrac{x}{2} \\ z = \dfrac{x}{3} \end{cases}$$

AHP cannot be applied on this example because of the form of the first preference, which is not a pairwise comparison.

If we solve this homogeneous linear system of equations as it is, we get x = y = z = 0, since its associated matrix is:

$$\begin{pmatrix} 1 & -2 & -3 \\ -1/2 & 1 & 0 \\ -1/3 & 0 & 1 \end{pmatrix} = -1 \neq 0$$

but the null solution is not acceptable since the sum x+y+z has to be 1.

Let's parameterise each right-hand side coefficient and get the general solution of the above system:

$$\begin{cases} x = 2\alpha_1 y + 3\alpha_2 z \\ y = \dfrac{\alpha_3}{2} x \\ z = \dfrac{\alpha_4}{3} x \end{cases}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0$.

Replacing the above second and third equations into the first one, we get

$$x = 2\alpha_1 \left(\frac{\alpha_3}{2} x \right) + 3\alpha_2 \left(\frac{\alpha_4}{3} x \right)$$

$$1 \cdot x = (\alpha_1\alpha_3 + \alpha_2\alpha_4) \cdot x$$

whence

$\alpha_1\alpha_3 + \alpha_2\alpha_4 = 1$ (parametric equation).

The general solution of the system is:

$$\begin{cases} y = \dfrac{\alpha_3}{2} x \\ z = \dfrac{\alpha_4}{3} x \end{cases}$$

whence one obtains the priority vector:

$$\left[x \quad \frac{\alpha_3}{2} x \quad \frac{\alpha_4}{3} x \right] \rightarrow \left[1 \quad \frac{\alpha_3}{2} \quad \frac{\alpha_4}{3} \right].$$

Fairness Principle: discount all coefficients with the same percentage: so, replace

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha > 0$ in the parametric equation we get $\alpha^2 + \alpha^2 = 1$, whence $\alpha = \frac{\sqrt{2}}{2}$.

Priority vector becomes: $\begin{bmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{6} \end{bmatrix}$

and normalizing it:

$$\begin{matrix} [0.62923 & 0.22246 & 0.14831] \\ C1 & C2 & C3 \\ x & y & z \end{matrix}$$

Preference will be on C1, the largest vector component. Let's verify it:

$\frac{x}{y} \cong 0.35354$ instead of 0.50, i.e. $\frac{\sqrt{2}}{2} = 70.71\%$ of the original.

$\frac{z}{x} \cong 0.23570$ instead of 0.33333, i.e. 70.71% of the original.

$x \cong 1.41421y + 2.12132z$ instead of $2y + 3z$, i.e. 70.71% of 2 respectively 70.71% of 3.

So, it was a fair discount for each coefficient.

7.2 Matrix Method of Using α -Discounting

The determinant of the previous homogeneous linear system is:

$$\begin{vmatrix} 1 & -2\alpha_1 & -3\alpha_2 \\ -\frac{1}{2}\alpha_3 & 1 & 0 \\ -\frac{1}{3}\alpha_4 & 0 & 1 \end{vmatrix} = (1+0+0) - (\alpha_2\alpha_4 + \alpha_1\alpha_3) = 0$$

or

$\alpha_1\alpha_3 + \alpha_2\alpha_4 = 1$ (parametric equation).

The determinant has to be zero in order for the system to have non-null solutions.

The rank of the matrix is 2.

So, we find two variables, for example it is easier to solve for y and z from the last two equations, in terms of x:

$$\begin{cases} y = \frac{1}{2}\alpha_3 x \\ z = \frac{1}{3}\alpha_4 x \end{cases}$$

and the procedure follows the same steps as in the previous one.

8 Strong Inconsistent Example 3

Let C = {C1, C2}, and P = {C1 is important twice as much as C2; C2 is important 5 times as much as C1}. Let m(C1)=x, m(C2)=y. Then:

x=2y and y=5x (it is a strong inconsistency since from the first equation we have x>y, while from the second y>x).

Parameterize: $x=2\alpha_1 y$, $y=5\alpha_2 x$, whence we get $2\alpha_1=1/(5\alpha_2)$, or $10\alpha_1\alpha_2=1$.

If we consider the Fairness Principle, then $\alpha_1= \alpha_2= \alpha>0$, and one gets $\alpha = \frac{\sqrt{10}}{10} \approx$ 31.62% consistency; priority vector is [0.39 0.61], hence y>x.

An explanation can be done as in paraconsistent logic (or as in neutrosophic logic): we consider that the preferences were honest, but subjective, therefore it is possible to have two contradictory statements simultaneously true since a criterion C1 can be more important from a point of view than C2, while from another point of view C2 can be more important than C1. In our decision-making problem, not having any more information and having rapidly being required to take a decision, we can prefer C2, since C2 is 5 times more important that C1, while C1 is only 2 times more important than C2, and 5 > 2.

If it's no hurry, more prudent would be in such dilemma to search for more information on C1 and C2. Or, we can set a threshold of consistency or inconsistency in order to accept or reject a result.

If we change Example 3 under the form: x=2y and y=2x (the two coefficients set equal), we get α = ½, so the priority vector is [0.5 0.5] and decision-making problem is undecidable.

9 Non-Linear/Linear Equation Mixed System Example 4

Let C = {C1, C2, C3}, m(C1)=x, m(C2)=y, m(C3)=z.

Let F be:

1. C1 is twice as much important as the product of C2 and C3.

2. C2 is five times as much important as C3.

We get the system: x=2yz (non-linear equation) and y=5z (linear equation).

The general solution vector of this mixed system is: $[10z^2\ \ 5z\ \ z]$, where z>0.

A discussion is necessary now.

a) You see for sure that y>z, since 5z>z for z strictly positive. But we don't see anything what the position of x would be?
b) Let's simplify the general solution vector by dividing each vector component by z>0, thus we get: $[10z\ \ 5\ \ 1]$.

If $z \in (0, 0.1)$, then y>z>x.
If z=0.1, then y>z=x.
If $z \in (0.1, 0.5)$, then y>x>z.
If z=0.5, then y=x>z.
If z>0.5, then x>y>z.

10 Non-Linear/Linear Equation/Inequality Mixed System Example 5

Since in the previous Example 4 has many variants, assume that a new preference comes in (in addition to the previous two preferences):

3. C1 is less important than C3.

The mixed system becomes now: x=2yz (non-linear equation), y=5z (linear equation), and x<z (linear inequality).

The general solution vector of this mixed system is: $[10z^2\ \ 5z\ \ z]$, where z>0 and $10z^2 < z$. From the last two inequalities we get $z \in (0, 0.1)$. Whence the priorities are: y>z>x.

11 Future Research

To investigate the connection between α-D MCDM and other methods, such as: the technique for order preference by similarity to ideal solution (TOPSIS) method, the simple additive weighting (SAW) method, Borda-Kendall (BK) method for aggregating ordinal preferences, and the cross-efficiency evaluation method in data envelopment analysis (DEA).

12 Conclusion

We have introduced a new method in the multi-criteria decision making, α- Discounting MCDM. In the first part of this method, each preference is transformed into a linear or non-linear equation or inequality, and all together form a system that is resolved – one finds its general solution, from which one extracts the positive solutions. If the system has only the null solution, or it is inconsistent, then one parameterizes the coefficients of the system.

In the second part of the method, one chooses a principle for finding the numerical values of the parameters {we have proposed herein the Fairness Principle, or Expert's Opinion on Discounting, or setting a Consistency (or Inconsistency) Threshold}.

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